

ON THE NUMBER OF COLOURS IN QUANTUM CHROMODYNAMICS

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Abstract

It is commonly believed that $\pi^0 \rightarrow 2\gamma$ decay shows that there are three colours in Quantum Chromodynamics (QCD). It is shown here that this is not correct. When correct colour dependent charges of the quarks are considered then it is shown that this decay does not make any statement about the number of colours in QCD.

It is important to know how many colours there are so that one may confidently formulate the theory of the strong interaction, ie Quantum Chromodynamics (QCD) around it. Fortunately there are several experimental indications that the number of colours in nature are three. In the authors view the best indication comes from the experimental evaluation of the R - ratio in the reaction

$$e^+ + e^- \rightarrow \text{hadrons} \quad (1)$$

However it is very often stated that even a more important proof of the number of colours being three comes from the experimental analysis of the reaction $\pi^0 \rightarrow 2\gamma$. Although this occurs in several places, for the sake of example here we would like to bring out this fact from references [1 - 4]. The aim of this short paper is to prove that this is not correct.

The decay $\pi^0 \rightarrow 2\gamma$ takes place through the anomaly in the divergence of axial vector currents [1 - 4]. The decay rate [1 - 4] is

$$\Gamma(\pi^0 \rightarrow 2\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_{\pi^0}^3}{64\pi^3 F_\pi^2} \quad (2)$$

where Q_u and Q_d are the u- and d- quark charges (in units of the proton charge), N_c is the number of colours, m_{π^0} is the neutral pion mass, $\alpha = \frac{e^2}{4\pi}$ and F_π , the pion decay constant is taken as 91 MeV.

The experimental value of the decay rate is around 7.48 eV. The quark charges are canonically taken as $Q_u=2/3$ and $Q_d=-1/3$ [1 - 4]. If there were no colours , ie $N_c = 1$ then from the above formula one gets the decay rate to be 0.84 eV. This is way off. Only when one takes the number of colours to be 3 in the above formula it is then that one obtains a satisfactory agreement with the experiment. This is then taken as a clean proof of three colours in QCD [1 - 4].

Now the factor

$$N_c^2 (Q_u^2 - Q_d^2)^2 \quad (3)$$

is the one which presumably determines the number of colours in the pion decay rate above. As shown above, for the u- and the d- quark charges 2/3 and -1/3 respectively, it is 1/9 when quarks are uncoloured and equal to one (and giving agreement with the experiment) when $N_c=3$.

However, in the above one has taken the u- and the d- quark electric charges to be static at 2/3 and -1/3 respectively. By the word 'static' we mean that they remain the same whether the number of colours were 1 or 3 or 5 or any other number. What we'd like to point out is that this is not correct. And this is the source of the problem here. As has been shown by the author, the electric charges of quarks have a colour dependence and the correct charges for the quarks are reproduced for three colours [5,6].

It has been shown by the author [5,6] that in the Standard Model when the $SU(N_c) \otimes SU(2)_L \times U(1)_Y$ (for $N_c = 3$) symmetry is spontaneous broken by a Higgs mechanism to $SU(N_c) \otimes U(1)_{em}$ symmetry and by maintaining all the built-in properties of the standard Model one finds the correct electric charges of the quarks as [5,6] :

$$Q_u = \frac{1}{2} \left(1 + \frac{1}{N_c} \right) \quad (4)$$

$$Q_d = \frac{1}{2} \left(-1 + \frac{1}{N_c} \right) \quad (5)$$

For $N_c = 3$ these reproduce the correct charges for the u- and the d- quarks as determined experimentally. That these are actually the correct charges and that they represent the correct colour dependence of the electric charge has been amply demonstrated by the author [5-9]. In fact this should be treated as a unique property and prediction of the Standard Model.

Note that as shown earlier it is the factor $N_c^2(Q_u^2 - Q_d^2)^2$ in expression 3 above in the decay rate of the pion which determined the number of colours to be three. However, with the correct colour dependent charges found by the author (equations 4 and 5 above) this factor is found to be always one (and thus giving good fit to the experiment), irrespective of the number of colours. This means that the decay rate of the pion gives good fit to the experiment for any number of colours. Thus clearly the decay rate of pion does not determine the number of colours. Hence, these are best determined by the other experimental methods as is well documented [1-4] but certainly not through the pion decay analysis (as shown here).

In short, we have shown that in contrast to what is commonly believed and which is well documented [1 - 4], the electric charges of the u- and the d- quarks are not just the static 2/3 and -1/3 respectively for any number of colours. The electric charges of the quarks have colour dependence [5 - 6]

and this is what has been used here. When these correct colour dependent charges are used for the decay rate of $\pi^0 \rightarrow 2\gamma$ then one finds that it has nothing to say about the number of colours in QCD. Hence the popular misconception as to this decay having anything to say about colour has to be rectified. The basic mistake that was made was not to realize that the electric charge, as arising in the Standard Model, has a colour dependence. This fact is important as QCD in the limit of N_c going to infinity continues to play a significant role for a proper understanding of the theory of the strong interaction.

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